

Large scale anisotropy due to pre-inflationary phase of cosmic evolution

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Abstract

We show that perturbations generated during the anisotropic pre-inflationary stage of cosmic evolution may affect cosmological observations today for a certain range of parameters. Due to the anisotropic nature of the universe during such early times, it might explain some of the observed signals of large scale anisotropy. In particular we argue that the alignment of CMB quadrupole and octopole may be explained by the Sachs-Wolfe effect due to the large scale anisotropic modes from very early times of cosmological evolution. We also comment on how the observed dipole modulation of CMB power may be explained within this framework.

1 Introduction

It is generally accepted that at early times, the universe might be anisotropic and inhomogeneous. The universe may evolve into a de Sitter space-time as it expands under the influence of a positive cosmological constant (Gibbons and Hawking 1977; Hawking and Moss 1982). Furthermore it has been shown (Wald 1983) that almost all Bianchi models asymptotically evolve into a de Sitter space in the presence of a positive cosmological constant. These are anisotropic but homogeneous models. The time scale for evolution to isotropy is $\sqrt{3/\Lambda}$, where Λ is the cosmological constant during inflation (Wald 1983). At time scales $t < \sqrt{3/\Lambda}$, the universe is anisotropic. Hence the modes which leave the horizon before this time are likely to show features of anisotropy.

An interesting observation of breakdown of isotropy in cosmological data is the alignment of quadrupole and octopole moments of CMBR (de Oliveira-Costa *et al* 2004; Ralston and Jain 2004; Eriksen *et al* 2004; Copi, Huterer and Starkman 2004; Schwarz *et al* 2004; Land and Magueijo 2005). One can define a preferred axis for both the quadrupole and octopole (de Oliveira-Costa *et al*; Ralston and Jain 2004), which point in the same direction to a very good

approximation. This axis is found to point towards ($l = 237.64^\circ, b = 62.95^\circ$), approximately towards the Virgo cluster of galaxies, and makes an angle of approximately 27° with the galactic axis (Aluri *et al* 2011). A possible explanation of this phenomenon is foregrounds, systematics or noise (Slosar and Seljak 2004; Abramo, Sodre and Wuensche 2006; Rakic, Rasanen and Schwarz 2006; Gruppuso, Burigana and Finelli 2007; Naselsky, Verkhodanav and Nielsen 2008). The possibility that foregrounds might cause the observed alignment has been ruled out (Aluri *et al* 2011), suggesting that its origin is most likely cosmological. It is very interesting that several other cosmological observations such as radio polarizations (Jain and Ralston 1999), Optical polarizations (Hutsemékers 1998) and the CMB dipole also points in the same direction (Ralston and Jain 2004). Another interesting observation is the dipole modulation of the CMB power (Eriksen *et al* 2004).

Some cosmological explanations for these anomalies include anisotropic inflation (Berera, Buniy and Kephart 2004; Gordon *et al* 2005; Ackerman, Carroll and Wise 2007; Erickcek, Carroll and Kamionkowski 2008; Kanno *et al* 2008; Yokoyama and Soda 2008; Koivisto and Mota 2008; Boehmer and Mota 2008), anisotropic/inhomogeneous spaces (Jaffe *et al* 2006, Land and Magueijo 2006, Bridges *et al* 2007; Ghosh, Hajian and Souradeep 2007; Pontzen and Challinor 2007; Kahniashvili, Lavrelashvili and Ratra 2008; Carroll, Tseng and Wise 2010) and local voids (Inoue and Silk 2006). It has also been suggested that initial phase of inflation, where the kinetic energy of the scalar field dominates, may explain some of these anomalies (Contaldi *et al* 2003; Donoghue, Dutta and Ross, 2009). Suitable estimators to characterise these primordial anisotropies have also been proposed (Hajian, Souradeep and Cornish 2004; Ralston and Jain 2004; Copi *et al* 2006; Bernui *et al* 2006; Armendariz-Picon 2006; Pullen and Kamionkowski 2007; Samal *et al* 2008; Groeneboom and Eriksen 2009; Bartolo *et al* 2011).

A mode with wave number \vec{k} leaves the horizon at times

$$k|\eta| < 1, \quad (1)$$

where $k = |\vec{k}|$ and the conformal time η is defined as

$$\eta(t) = \int_{t_e}^t \frac{dt'}{a(t')}, \quad (2)$$

with t_e equal to the time at the end of inflation. Note that by this definition η is negative at times before the end of inflation and positive later on. During

inflation, the universe undergoes a rapid phase of expansion with the scale factor growing as

$$a(t) = a_I e^{H_I t}, \quad (3)$$

where H_I is the Hubble constant during inflation. This gives

$$\eta = -\frac{1}{H_I a(t)} \left[1 - \frac{a(t)}{a(t_e)} \right] \approx -\frac{1}{H_I a(t)}, \quad (4)$$

where we ignore the contribution from t_e since $t_e \gg t$. Throughout this paper we shall assume a spatially flat FRW metric, after the universe becomes isotropic.

During inflation, the curvature scalar $R = 12H_I^2$. Hence we find that $\Lambda \sim 12H_I^2$. This implies that the time, t_{iso} , after which isotropy sets in is of order $t_{iso} \sim 0.5/H_I$. Using Eq. 1, we find that at these early times only modes with wave number,

$$k < H_I a_I e^{0.5}, \quad (5)$$

leave the horizon. If these modes, which are generated during the anisotropic phase, before the universe evolves into a de-Sitter space-time, re-enter the horizon before the current era, then these could lead to large scale anisotropy in cosmological observations.

We may assume that the anisotropy causes all these wave vectors to lie in a plane and that this plane is perpendicular to the axis of alignment of the CMB quadrupole and octopole (de Oliveira-Costa 2004, Ralston and Jain 2004, Copi, Huterer and Starkman 2004, Schwarz *et al* 2004, Land and Magueijo 2005). We shall refer to this axis as the preferred axis and the plane perpendicular to it as the preferred plane. Hence these anisotropic modes will lead to fluctuations in the metric which lie in a plane. Such anisotropic metric fluctuation could lead, directly or indirectly, to some of the claimed signals of large scale anisotropy.

The anisotropic metric fluctuations could lead to alignment of the low l CMB multipoles through the Sachs-Wolfe effect (Sachs and Wolfe 1967; Hu and Sugiyama 1995; Francis and Peacock 2010, Dodelson 2003, Durrer 2008, Weinberg 2008). The photons which propagate perpendicular to the preferred plane do not experience any gravitational potential. Hence these do not get any contribution due to the Sachs-Wolfe effect. However the photons that propagate along the plane experience maximum effect since the metric fluctuations are maximal in this plane. This will lead to additional anisotropy in the CMB temperature fluctuations. If this effect is sufficiently strong it

might lead to alignment of low l multipoles. We investigate this possibility in the present paper.

2 Anisotropic metric perturbations at large distance scales

The anisotropic perturbations, which are generated during the early anisotropic phase of cosmic evolution, may re-enter the horizon at late times. A perturbation that leaves the horizon at conformal time $\eta = -1/k$, re-enters the horizon at time $\eta = 1/k$. In this section we determine the conformal time at which the primordial perturbations zoomed to super horizon scales during the early time of inflation, re-enter the horizon.

The conformal time before the end of inflation is given by Eq. 4. At the end of inflation $\eta = 0$. We assume that, immediately after that radiation dominates the energy density of the universe. The scale factor during this phase may be expressed as,

$$a(t) = At^{1/2}. \quad (6)$$

We can fix A by equating this to the inflationary solution at time t_e corresponding to the end of inflation. We find that during the radiation domination phase,

$$\eta = \frac{2a(t)t_e}{a^2(t_e)} \left[1 - \frac{a(t_e)}{a(t)} \right]. \quad (7)$$

We point out that the time t_e may be fixed by requiring that the number of e-folds the universe has expanded into, during inflation, are $N \approx 64$. Hence we require,

$$H_I t_e = N \approx 64. \quad (8)$$

Hence we find that the conformal time at the radiation matter equality is equal to,

$$\eta_{eq} \approx \frac{2a_{eq}}{a^2(t_e)} \frac{N}{H_I}, \quad (9)$$

where $a_{eq} = a(t_{eq})$.

We next compute the conformal time during matter domination phase. We assume that matter dominates for time $t > t_{eq}$ and ignore all other contributions. Here we ignore the contribution due to late time acceleration

also, since that will only lead to a small correction to our results. The scale factor during this phase may be expressed as,

$$a(t) = Bt^{2/3}. \quad (10)$$

We find the conformal time during this phase to be,

$$\eta = 3 \sqrt{\frac{a(t)a_{eq}}{a^2(t_e)}} \frac{N}{H_I a(t_e)} \left[1 - \sqrt{\frac{a_{eq}}{a(t)}} \right] + \eta_{eq}. \quad (11)$$

Hence at late times, we find,

$$\eta(t) \approx \frac{3N}{H_I a(t_e)} \sqrt{\frac{a(t)a_{eq}}{a^2(t_e)}}. \quad (12)$$

Let t_l represent the time when a mode leaves the horizon during inflation. The time t_r , when it re-enters the horizon, is given by,

$$\frac{a(t_e)}{a(t_l)} \approx 3N \frac{a_{eq}}{a(t_e)} \sqrt{\frac{a(t_r)}{a_{eq}}}. \quad (13)$$

Here we have assumed that the mode re-enters the horizon during matter domination.

We next make an estimate of t_r in order to determine whether the anisotropic modes might have some effect on the cosmological observations made today. We are interested in computing the time corresponding to the modes which leave the horizon when the universe has just entered the de-Sitter phase. All the modes which leave the horizon before this time are generated during the anisotropic phase of evolution. Hence, in our case, the time t_l corresponds to the very early time during inflation. Since inflation lasts approximated 28 efolds, we expect,

$$\frac{a(t_e)}{a(t_l)} \approx 10^{28}. \quad (14)$$

Thus we get,

$$\frac{a(t_r)}{a_{eq}} = 7 \times 10^{-3} \left[\frac{10^{19} \text{GeV}}{H_I} \right] \left[\frac{5 \times 10^4 \text{yr}}{t_{eq}} \right] \left[\frac{a(t_e)/a(t_l)}{10^{28}} \right]^2 \left[\frac{64}{N} \right]. \quad (15)$$

This equation gives the scale factor, $a(t_r)$, when the modes which left the horizon when the universe crossed over from anisotropic to de Sitter phase,

re-entered the horizon. It shows that there exists allowed parameter range where the anisotropic modes, generated before the phase of isotropic and homogeneous inflation, can re-enter the horizon before the current time and hence have observational consequences. To the best of our knowledge, this simple and basic result, Eq. 15, does not exist in the literature so far. We find that if H_I is of the order of the Planck scale then the modes generated during the anisotropic phase enter the horizon during radiation dominated phase, i.e. before decoupling. Hence the large wavelength modes will show signals of statistical anisotropy. These modes may therefore explain the anomalies observed at low l . If H_I is of order 10^{15} GeV these modes enter the horizon during the matter dominated phase. In this case the anisotropic modes will have no contribution at the time of decoupling of radiation from matter. However as these anisotropic modes enter the horizon at late times, they affect the low l multipoles through integrated Sachs-Wolfe (ISW) effect (Sachs and Wolfe 1967; Hu and Sugiyama 1995; Francis and Peacock 2010, Dodelson 2003, Durrer 2008, Weinberg 2008).

In Fig. [1] we schematically show a modified cosmic history, assuming that the parameters of inflation are such the anisotropic modes enter the horizon before the current era. The corresponding parameter space is given by Eq. 15. In Fig. [1] the time t_i corresponds to an early time when the universe starts evolving under the influence of the cosmological constant. The universe is anisotropic at this time. It becomes isotropic at time equal to t_{iso} . The standard inflationary phase starts at this time. The anisotropic modes which leave the horizon at time t_l re-enter the horizon at t_r . The precise value of t_r depends on the inflationary parameters, as governed by Eq. 15. Hence the anisotropic modes generated before t_l re-enter the horizon after t_r and can affect cosmological observations if $t_r < t_0$, where t_0 is the current time.

We emphasize that if the parameters are such that these anisotropic modes re-enter the horizon at a sufficiently early time, then they can in principle explain the wide range of anisotropic signals observed. These include the large scale anisotropies seen in CMB (de Oliveira-Costa *et al* 2004; Ralston and Jain 2004; Eriksen *et al* 2004; Copi, Huterer and Starkman 2004; Schwarz *et al* 2004), optical polarizations (Hutsemékers 1998; Agarwal *et al* 2011), radio polarizations (Jain and Ralston 1999), coherent flow in cluster peculiar velocities (Kashlinsky *et al* 2010) and dipole anisotropy in galaxy distribution (Itoh *et al* 2010). This is due to the wide range of possible anisotropic models which might be applicable at early times. Furthermore

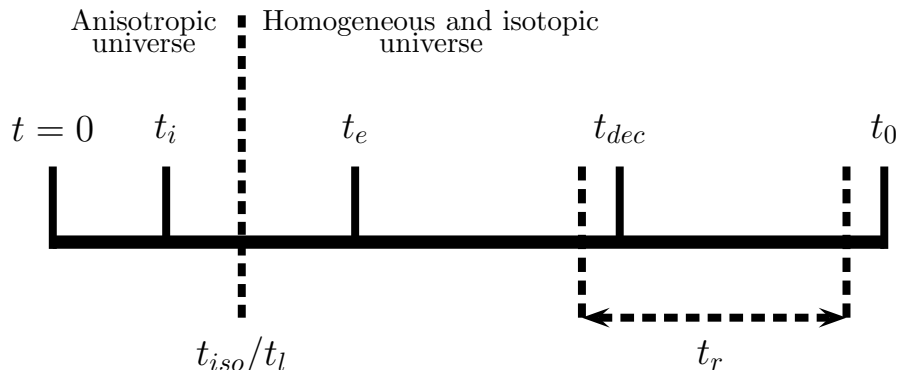


Figure 1: A schematic illustration (not to scale) of the various time scales mentioned in this paper. The universe evolves anisotropically under the influence of cosmological constant at time t_i . The isotropic inflationary phase starts beyond the time scale t_{iso} . Inflation ends at time t_e and t_{dec} denotes the time scale of decoupling. For a certain range of allowed parameters, the anisotropic modes which leave the horizon before t_l may re-enter the horizon after t_r such that $t_r < t_0$, where t_0 is the current time. Hence these can affect cosmological observables.

the anisotropic nature of the modes created at these early times is preserved over much of the history of cosmic evolution. This is because as long as these modes are outside the horizon, no causal physics can affect them. Only after they re-enter the horizon do they start evolving significantly. However at this stage they also start affecting cosmological observables. It would clearly be of great interest to explore a wide range of these anisotropic models to determine if they can explain the observed violations of isotropy.

3 Low l CMBR multipoles

As discussed in section 2, we find, using Eq. 15, that there exists a wide range of parameters for which the modes generated during the early anisotropic phase of cosmic evolution may play a role in the present day cosmology. This applies to a wide class of models, discussed in Wald (1983), which evolve into de Sitter space-time. Due to the large range of allowed models, which fall in this class, it is possible that a model may exist which might explain the anisotropic signals observed in CMBR. Here we discuss a simple illustrative

example of how these anisotropic modes might lead to the observed low l anomalies. Here we are not interested in detailed metric models. We shall assume a simple anisotropic model of perturbations with the preferred direction aligned along the z axis. Hence we shall assume that the modes with wave vectors aligned along the z axis behave differently from those which lie in the $x - y$ plane. Assuming some reasonable properties of these modes we determine if they can explain alignment of the low l multipoles.

The anisotropic modes leave the horizon at very early times. We assume that the parameters are chosen such that they re-enter the horizon before the time of decoupling. As these anisotropic modes re-enter the horizon, they can affect the CMBR photons. We assume that the wave vectors corresponding to these perturbations all lie in a plane which is perpendicular to the Virgo axis which is roughly the axis of alignment of CMBR quadrupole and octopole. We refer to this axis and the corresponding plane as the preferred axis and preferred plane respectively. We shall choose our coordinates such that this anisotropy axis points towards \hat{z} . Photons propagating perpendicular to this plane would be unaffected by the anisotropic metric perturbations. However the photons propagating in other directions would undergo redshift or blueshift due to the Sachs-Wolfe effect. This effect would be maximum for photons propagating parallel to the preferred plane. Hence it would induce additional anisotropies in the CMBR spectrum which would lie dominantly in the preferred plane. If these have sufficient strength, then they may yield a preferred axis for quadrupole and octopole, perpendicular to this plane. Hence this phenomenon can explain the alignment of $l = 2, 3$ multipoles.

Let's first assume that these modes, generated during the early anisotropic phase of cosmic evolution, enter the horizon just before decoupling. Let $\psi(\eta_{dec}, \vec{x}_{dec})$ denote the gravitational potential. Here η_{dec} is the conformal time during decoupling, $\vec{x}_{dec} = \vec{x}(\eta_{dec})$ and

$$\vec{x}(\eta) = \vec{x}_0 - (\eta_0 - \eta)\hat{n}, \quad (16)$$

where \vec{x}_0 is the position of the observer, η_0 the conformal time today, and \hat{n} is the direction of observation. Ignoring the integrated Sachs-Wolfe effect we find,

$$\left(\frac{\Delta T(\hat{n})}{T} \right) \sim \psi(\vec{x}_{dec}, \eta_{dec}). \quad (17)$$

We note that here we have assumed the formula corresponding to adiabatic perturbations. This captures the basic physics that the anisotropies are related to the metric perturbations. However the detailed final result is likely

to be dependent on the precise anisotropic model that may be applicable during the early phase of cosmic evolution. Hence we do not try to predict the absolute magnitude of these temperature anisotropies. We assume that by suitable choice of parameters these can be adjusted to fit the data. Furthermore here we have ignored the contribution due to the statistically isotropic modes. We assume that these are negligible for low l and their contribution increases as we increase l . Hence for low l the anisotropic modes dominate whereas these are negligible for higher l . A more detailed treatment is postponed to future research.

We express the potential in terms of its fourier transform $\tilde{\psi}(\vec{k}, \eta)$,

$$\psi(\vec{x}, \eta) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} \tilde{\psi}(\vec{k}, \eta). \quad (18)$$

We next model $\tilde{\psi}(\vec{k}, \eta)$ as

$$\tilde{\psi}(\vec{k}, \eta) = 2\pi\delta(k_z)g(\vec{k}_\perp, \eta), \quad (19)$$

where \vec{k}_\perp is the projection of the wave vector in the $x - y$ plane. This implements our idea that all the modes lie in the $x - y$ plane. It may of course be useful to replace the delta function by a function strongly peaked at $k_z = 0$. However in the present paper we use only this extreme model where all the modes lie strictly in the $x - y$ plane. We denote the magnitude $|\vec{k}_\perp| = k_\perp$. We assume that

$$g(\vec{k}_\perp, \eta) \propto 1/k_\perp, \quad (20)$$

up to a cutoff $k_\perp = k_c$. Beyond this cutoff the anisotropic modes do not contribute. The anisotropy spectrum generated by these low \vec{k} modes may be expressed as,

$$\frac{\Delta T(\hat{n})}{T} = \frac{k_c^2}{2\pi} \int_0^1 y dy J_0(ay) g(k_c y), \quad (21)$$

where we have defined the variable $y = k_\perp/k_c$. The symbol a is given by

$$a = \sqrt{(k_c x_{0\perp})^2 + (k_c \Delta\eta)^2 n_\perp^2 - 2(k_c \Delta\eta)(k_c \vec{x}_{0\perp}) \cdot \vec{n}_\perp}, \quad (22)$$

where $\Delta\eta = \eta_0 - \eta_{dec}$ and $\vec{x}_{0\perp}$ is the component of the vector \vec{x}_0 in the $x - y$ plane, perpendicular to the preferred direction.

We assume, without loss of generality, that $\vec{x}_{0\perp}$ points in the x direction. This simply corresponds to a choice of axes. If we set this vector to $\vec{0}$ then the resulting temperature fluctuations depend only on $|\vec{n}_\perp|$, which is uniform in the $x - y$ plane. Hence in this case the temperature is same every where in the preferred plane. In order to generate fluctuations in this plane we need another vector such as $\vec{x}_{0\perp}$. Alternatively we may assume that the function $g(\vec{k}_\perp, \eta)$ doesn't just depend on the magnitude of \vec{k}_\perp . In this case also, we shall essentially need to introduce another vector in the $x - y$ plane. In the present paper, we assume that $\vec{x}_{0\perp}$ is not equal to zero and that $g(\vec{k}_\perp, \eta)$ depends only on the magnitude of \vec{k}_\perp .

In Fig. [2], we show the resulting temperature fluctuations, in arbitrary units, as a function of the position on the sky. Here we have set $k_c|\vec{x}_{0\perp}| = 6$ and $k_c\Delta\eta = 3$. We speculate that the second preferred axis in the $x - y$ plane may be the axis of the ecliptic dipolar power asymmetry (Eriksen *et al* 2004). It lies in a plane perpendicular to the axis pointing towards Virgo, in approximate agreement with observations. Thus, this simple model studied here may simultaneously account for both the axis of anisotropy seen in the CMB data.

It is clear from the figure that the hot spot of all the modes is aligned along one direction, which in our coordinate system is the x -axis. Furthermore if we compute the principal axis (Ralston and Jain 2004, Samal *et al* 2008), we find that the axes for $l = 2, 3, 4, 5, 6$ point towards the z -axis for this map. The principal eigenvectors (PEV) and the corresponding power is shown in Table 1. We find that the power due to these anisotropic modes decreases with increase in l . In fact, for our choice of parameters, the power in multipoles $l \geq 4$ is negligible compared to the multipoles $l = 2, 3$. Hence it is reasonable to assume that the multipoles $l \geq 4$ may receive dominant contributions from statistically isotropic modes. Hence the SW effect considered here may lead to alignment only for the multipoles $l = 2, 3$ for our choice of parameters. It is clear, however, that this is model dependent and results will change depending on our choice of parameters. Observations show dominant alignment only between the quadrupole and octopole. However, there is also some evidence that this alignment might continue to larger l (Samal *et al* 2008; Samal *et al* 2009).

Multipole, l	Power, $l(l+1)C_l/2\pi$ (in arbitrary units)	PEV (x,y,z)
2	0.024	(-4.765E-8, 1.423E-8, 0.9999)
3	0.014	(-5.424E-8, -2.722E-9, 0.9999)
4	0.0018	(-1.767E-7, -7.465E-9, 0.9999)
5	0.00016	(-3.399E-7, 4.459E-8, 0.9999)
6	0.0000058	(1.322E-6, -1.992E-8, 0.9999)

Table 1: Principal eigenvectors (PEVs) of the CMB signal shown in Fig. [2] due to SW effect from pre-inflationary anisotropic modes.

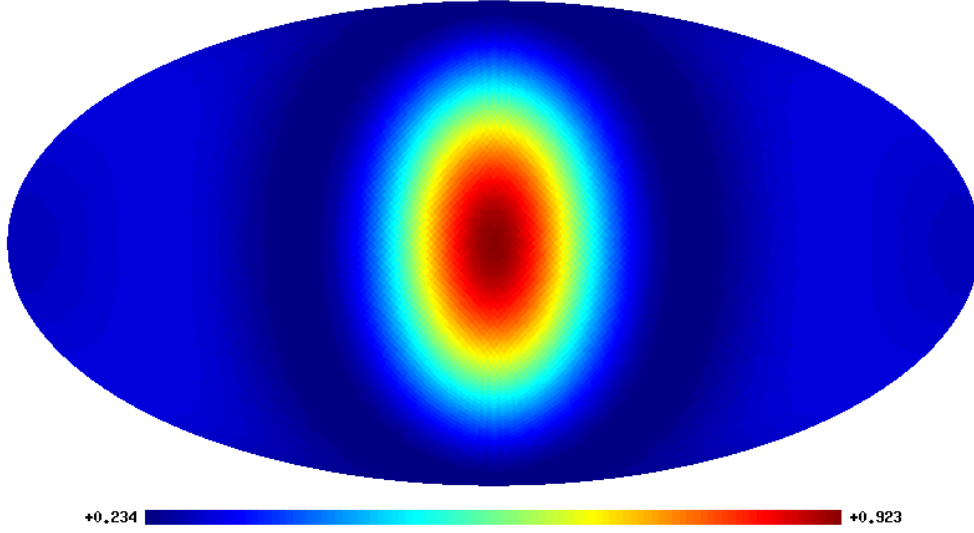


Figure 2: The temperature anisotropy generated by the anisotropic modes due to Sachs-Wolfe effect. The map is generated at HEALPix resolution of $N_{side} = 32$.

4 Conclusions

In this paper, we study the implications of anisotropic primordial perturbations, carrying imprints of pre-inflationary anisotropic era, to the large scale anomalies found in CMB. We have shown that the modes which leave the horizon during the early anisotropic phase re-enter the horizon before the current time for a wide range of choice of the Hubble parameter during inflation. For a certain range of allowed values of the Hubble parameter during inflation, these may enter the horizon even before decoupling. Hence these can provide an explanation of some of the large scale anomalies of CMB through Sachs-Wolfe effect (or integrated Sachs-Wolfe effect). We have described a simple illustrative model of these anisotropic modes which shows alignment of the low l multipoles. For an appropriate choice of parameters of the model, we have shown that the contributions due to the SW effect is such that they may cause alignment of quadrupole and octopole in the observed signal. The model requires another preferred axis in the plane perpendicular to the principal axis of quadrupole and octopole. We speculate that this second axis might be related to the dipole modulation axis discovered in (Eriksen *et al* 2004).

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